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FINITE ELEMENT CALCULATIONS OF CYLINDER NATURAL FREQUENCIES

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Abstract

Computer programs have been developed at DREA for predicting natural frequencies of structures submerged in a dense compressible fluid. This technical communication presents results from numerical studies performed to assist in the validation of these programs. Two unstiffened right cylinders are analysed using the finite element codes COUPLE and VAST and the results are compared with published results from other finite element codes, analytical results, and experimental data.

Résumé

Le CRDA a développé des programmes informatiques pour prédire la fréquence propre des structures submergées dans un fluide compressible dense. Cet article technique présente des résultats obtenus à partir d'études numériques effectuées pour faciliter la validation de ces programmes. Deux cylindres droits non raidis sont analysés à l'aide des codes d'éléments finis COUPLE et VAST et les résultats sont comparés à ceux fournis par d'autres codes d'éléments finis, de résultats analytiques et de données expérimentales.

Contents

1 Introduction

Computer programs have been developed at DREA for predicting natural frequencies of structures submerged in a dense compressible fluid. The codes involved are a general purpose finite element package, VAST [1], and COUPLE [2, 3, 4], a program for establishing a fluid finite element model and coupling it to the structural model.

These codes have been verified to some degree, but further examples for comparison are always useful and eagerly sought. Two articles recently came to the author's attention which contain examples of numerical and experimental results from submerged cylinder tests which would be of use as test cases against which the performance of the VAST/COUPLE combination could be measured. A paper by G.C. Everstine, "Prediction of Low Frequency Vibrational Frequencies of Submerged Structures" [5], discusses a NASTRAN-based method for predicting natural frequencies and gives results *in vacuo* and submerged for axisymmetric and shell models of an unstiffened right cylinder. A paper by W.G. Price, *et al.*, "Fluid-Structure Interaction of Submerged Shells" [6], predicts *in vacuo* natural frequencies of an unstiffened right cylinder with a finite element method and submerged frequencies with an analytic method based on the *in vacuo* resonances. This paper also presents experimental results done both in air and submerged for the same model.

In this report, the results from these papers will be presented side-by-side with results determined using the VAST/COUPLE combination of in-house codes.

2 Everstine Comparison

In [5], Everstine outlines a method for using NASTRAN and other codes, in much the same way that VAST and COUPLE are used, to determine the natural frequencies of a submerged structure. The paper discusses both a boundary element approach and a finite element approach to modelling the fluid. In the paper, a finite element model of one half of a cylinder is generated using both axisymmetric elements and regular shell elements. The finite element approach is used to model the fluid in the axisymmetric case and the boundary element approach is used in the three-dimensional case.

2.1 Cylinder Specifications

The model tested by Everstine was a right cylinder with an aspect ratio (length to diameter) of six to one. The dimensions for the model are given in Table 1.

In [5], the axisymmetric model of the structure was generated using two-noded shell elements. Only one half of the structure was modelled with 4 elements on the end cap and 25 along the half-length. Five layers of triangular ring elements to a distance of 16 metres from the shell's axis (a fluid thickness of 2.2 times the shell radius) were used to model the fluid.

Dimension	Value
Length	60.0 m
Diameter	10.0 m
Wall Thickness	0.05 m
Endcap Thickness	0.05 m
Young's modulus	196 GPa
Poisson's ratio	0.3
Material density	7900 kg/m^3
Fluid density	1000 kg/m^3
Fluid sound speed	1500 m/s

Table 1: Particulars of Everstine Cylinder Model

The NASTRAN QUAD4 element was used in the shell model of the cylinder with 4 elements radially, 12 elements over the half circumference, and 25 elements along the half-length (in total, a quarter model). Boundary elements matching the shell elements were used to model the fluid.

Note that due to the partial models used, only modes symmetric with respect to the mid-length were calculated; i.e., axial wave numbers (M) are all odd.

2.2 Results

2.2.1 *In Vacuo*

The VAST axisymmetric model (see Figure 1) was meshed to match the Everstine model with four elements along each end and fifty elements along the full length. Using three-noded axisymmetric shell elements, this resulted in a model with 58 elements and 117 nodes (assigned prefix EVER).

The three-dimensional eight-noded shell element model was meshed with 36 elements on each end, 24 elements around the circumference, and 18 elements along the length. With the eight-noded shell element, this resulted in a model with 504 elements and 1514 nodes (see Figure 2) which was assigned the prefix EVER2.

A similar model was constructed using four-noded shell elements. This was done to allow the construction of the fluid model using infinite fluid elements. COUPLE has an eight-noded infinite fluid element, but not a twenty-noded element. This model was meshed with 64 elements on each endcap, 32 elements circumferentially, and 18 elements axially resulting in a total of 706 nodes and 704 elements. The model (prefix EVER5) is shown in Figure 3.

These three models were run through VAST to determine the in-air natural frequencies and the results are shown in Table 2 where the heading VAST8 represents the 8-noded shell

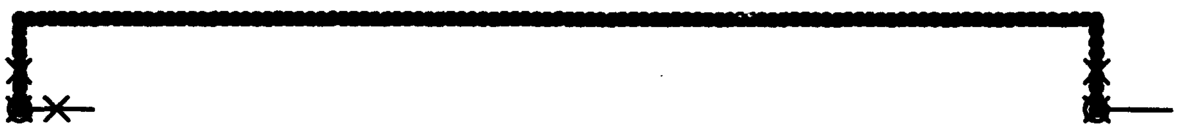

Figure 1: VAST Axisymmetric Model (Everstine)

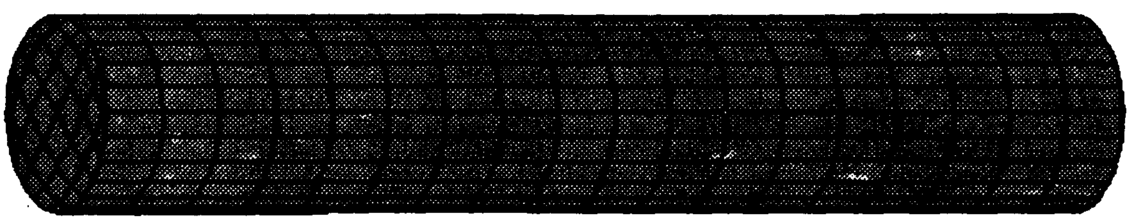

Figure 2: VAST 8-Noded Shell Element Model (Everstine)

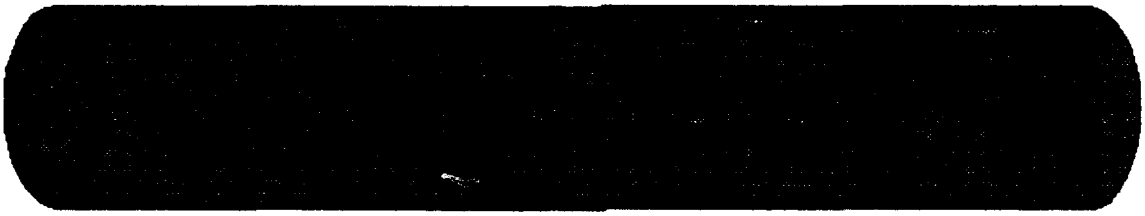

Figure 3: VAST 4-Noded Shell Element Model (Everstine)

element model and VAST4 the 4-noded shell model. The natural frequencies of the two three-dimensional models were calculated with the models constrained in a quasi free-free state. This quasi free-free state is such that the cylinder is fully constrained in all six degrees-of-freedom, but this is done in such a way that the resonant modes are virtually indistinguishable from those determined in a free-free analysis. The centers of both endcaps are constrained against radial translations and additional nodes close to the centers are constrained in one of the two coordinate directions to prevent rigid-body rotations. To prevent axial translations, some nodes on the perimeter at the half length are constrained against axial motion.

In Table 2, and all subsequent tables, the circumferential wave numbers are indicated by N and the axial wave half-numbers by M. To conform to the convention used by Everstine, the modes involving the end caps have a third indicator, M_p, added. The modes with a value for M_p are explained in Everstine's paper; however, the explanation is less than clear and, as such, there was some confusion in assigning calculated eigenvalues to the tables.

The use of full models was dictated by the COUPLE program (the fluid model), where the capability for defining planes of symmetry has yet to be implemented. As such, many more intermediate modes (the even modes) were calculated with VAST than in the Everstine analyses. As only roughly the same number of modes were calculated, this results in the lists under the VAST headings in the table appearing incomplete.

Table 2 shows, that except for the endcap modes, the VAST predictions are virtually identical to Everstine's. The differences in the endcap modes are large. There appears to be no obvious explanation for the differences, although the confusing notation used by Everstine may have led to a mismatch of some modes. Some endcap modes predicted by Everstine could not be identified at all, as it was not clear what they were. The VAST model which used the 4-noded elements did predict higher values, but the differences were acceptable for the first few modes. This model had only one half the number of nodes as the other VAST model, and as such, was not expected to perform as well, but was deemed acceptable for the coupled analysis.

Harmonic			Axisymmetric		Three-Dimensional		
N	M	M_p	Ever.	VAST	Ever.	VAST8	VAST4
2	1		2.72	2.72	2.72	2.72	2.80
3	1		3.84	3.84	3.90	3.85	4.02
0		1	4.27	4.89	4.22	4.88	5.11
4	1		7.04	7.04	7.19	7.08	7.62
4	3		9.29	9.23	9.34	9.27	10.3
1		1	9.53	6.67	9.20	10.8	11.3
3	3		10.4	10.3	10.4	10.3	11.4
5	1		11.3	11.3	11.6	11.4	12.8
5	3		12.2	12.2	12.4	12.3	13.9
1	3		13.4	12.9	13.3	–	–
2		1	15.6	12.9	15.1	–	–
5	5		15.8	15.7	15.9	–	–
0		2	15.9	20.6	16.4	–	–
4	5		17.0	16.7	16.9	–	–
2	3	3	18.6	?	18.5	–	–
3		1	22.7	24.1	22.4	–	–
5	7		22.9	22.4	22.8	–	–
3	5		24.5	24.0	24.1	–	–
1	0	2	26.6	?	27.2	–	–
4	7		28.4	27.7	28.2	–	–

Table 2: Natural Frequencies of Everstine Cylinder Model (*in vacuo*)

2.2.2 Submerged

Natural frequencies for the submerged cylinder were determined using the COUPLE/VAST combination. The axisymmetric fluid model used four layers of 8-noded axisymmetric elements for a total fluid thickness of 20 m (fluid model file name EVEF). The coupled system had a total of 117 structural nodes and 821 fluid nodes (file name EVEC).

The shell fluid model used one layer of 8-noded brick elements and one layer of 8-noded infinite elements. The infinite elements used a $\frac{1}{r}$-type decay function with a decay length, l_d, of 3.0. The fluid model (file name EVERF) thus had a total of 2118 nodes. The coupled system (file name EVERC) had a total of 5648 DOF.

The results of the COUPLE/VAST analysis are shown in Table 3. Unfortunately, the coupled analysis could not predict the same modes as did the in-air analysis. The VAST program can allow for solution of the symmetric and non-symmetric harmonics while COUPLE can only deal with the in-plane modes. Thus, for such a model, COUPLE can only predict endcap and interframe bending modes. This is seen in the table, where there is only one entry for the VAST axisymmetric analysis. While of limited use, the model did match the Everstine prediction.

There are fairly large discrepancies between the NASTRAN and VAST results for the shell model of the cylinder. While the frequency values follow similar trends, VAST predicts higher frequencies throughout, varying from ten to forty percent higher. As there are no experimental results against which to evaluate either one and the two models are virtually identical, it is not possible to determine if one is more correct than the other, although it should be noted that the VAST axisymmetric model result matches Everstine's results.

3 Price Comparison

Price, *et al.*, in [6] describe a study in which experimental, analytical, and numerical work were combined in the analysis of an unstiffened steel cylinder. The experimental tests involved measuring the natural frequencies of a horizontal, freely suspended cylinder in air and in a tank of water. The natural frequencies were then predicted (*in vacuo*) by finite element methods and, using these resonances as a basis, the submerged frequencies were calculated using an analytical approach.

3.1 Cylinder Specifications

The model tested was also right cylinder, but of lower aspect ratio. The dimensions for the model are given in Table 4. The material properties are assumed, since the actual values were not given in Reference [6]. For the tank test, the cylinder was submerged so that its longitudinal axis was 680 mm below the free water surface. The tank was rectangular of length

Harmonic			Axisymmetric		Three-Dimensional	
N	M	M_p	Ever.	VAST	Ever.	VAST4
2	1		1.13	–	1.13	1.25
0		1	1.63	1.61	1.44	2.01
3	1		1.79	–	1.81	2.06
4	1		3.61	–	3.67	4.45
1		1	4.44	–	4.26	6.01
4	3		4.81	–	4.82	6.05
3	3		4.94	–	4.93	5.96
5	1		6.31	–	6.38	–
5	3		6.83	–	6.86	–
1	3		7.04	–	6.88	–
2		1	8.31	–	8.02	–
0		2	8.66	–	8.40	–
5	5		8.99	–	8.88	–
4	5		8.94	–	8.85	–
2	3	3	8.05	–	8.07	–
3	5		11.9	–	11.9	–
3		1	13.2	–	13.0	–
5	7		13.2	–	12.9	–
1		2	15.4	–	15.5	–
4	7		15.3	–	15.1	–

Table 3: Natural Frequencies of Everstine Test Cylinder (in water)

Dimension	Value
Length	1284 mm
Diameter	360 mm
Wall Thickness	3 mm
Endcap Thickness	3 mm
Young's modulus	200 GPa
Poisson's ratio	0.3
Material density	7600 kg/m^3
Fluid density	1000 kg/m^3
Fluid sound speed	1500 m/s

Table 4: Particulars of Price Cylinder Model

7.5 m, width 1.7 m, and depth 1.6 m and the cylinder was placed across the tank's width. Three finite element models were examined. The first used a thin shell of revolution (axisymmetric) element (3-noded), the second a flat facet shell element (8-noded), and the third a curved shell element (8-noded). This last model is illustrated in [6]. It is meshed with 3 elements radially, 16 circumferentially, and 18 axially. The results for these analyses, shown in Table 5, are averages of the results from [6] for these three types of models. The analytic results, shown in Table 6, were calculated for water depths in the tank of 1.6 m and infinity.

3.2 Results

3.2.1 *In Vacuo*

Two models were tested for comparison using VAST. The first (file name PRIC1) used 8-noded shell elements for close comparison to the Price model. The model endcaps were meshed with a 6×6 grid while the rest of the shell was modelled with 24 elements circumferentially and 18 axially resulting in a total of 1514 nodes and 504 elements (see Figure 4). The second model (PRIC8) was set up for the requirements of COUPLE, as above, using 4-noded shell elements. This model had 64 elements on each endcap, 32 elements circumferentially and 12 elements axially. This mesh resulted in a total of 514 nodes and 512 elements (see Figure 5).

The natural frequencies of the PRIC1 model in the free-free state are given in Table 5 while the PRIC8 model resonances were calculated in the quasi free-free state to meet the fluid model requirements. The first model is indicated by FE8, the second by FE4.

Many of the modes calculated show up in the analysis as two distinct values (the orthogonal set). In these cases, the number shown in the table is based on averages of these two values.

Figure 4: VAST 8-Noded Element Model (Price)

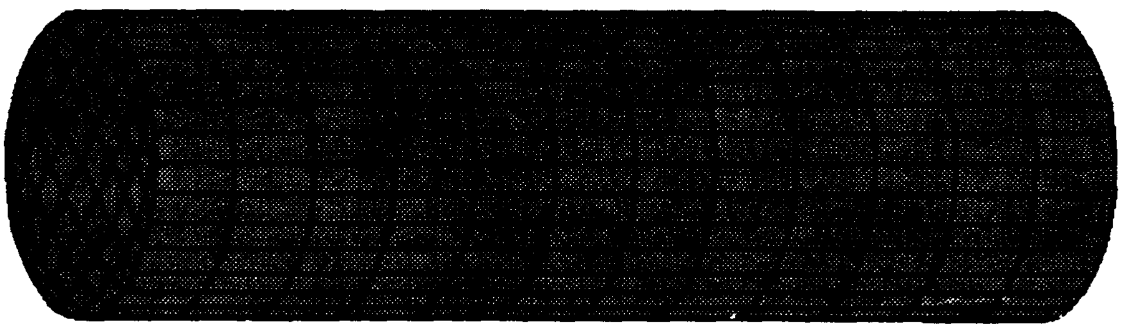

Figure 5: VAST 4-Noded Element Model (Price)

Harmonic		Price		VAST	
N	M	FE	Exp.	FE8	FE4
2	1	198	194	196	202
3	1	204	198	199	208
end	cap	209	–	220	235
end	cap	217	–	228	240
4	1	354	336	341	367
3	2	404	387	388	439
4	2	–	403	405	439
5	1	–	537	546	611
4	3	–	565	570	655
5	2	–	–	575	643

Table 5: Natural Frequencies of Price Test Cylinder (*in vacuo*)

The worst error in 8-noded element model is less than two percent (N=5, M=1) and the values are more accurate than those predicted by Price. The worst error in FE4 is about 16 percent (N=4,M=3). This could be improved with a finer grid; however, this was felt to be acceptable accuracy for this analysis.

3.2.2 Submerged

Natural frequencies for the submerged cylinder were determined using the COUPLE/VAST combination starting with the structural model utilizing the 4-noded elements (to allow for the use of COUPLE's 8-noded infinite brick element). Two fluid models were constructed. One model represents an infinite fluid using one layer of 8-noded infinite elements. These elements used a $\frac{1}{r}$-type decay with a decay length, l_d, of 1.25. This model (file name PRICI) has a total of 1028 fluid DOF. The results of the coupled system (file name PRICA) are shown in Table 6 under the infinite depth VAST heading. The other fluid model (file name PRICF) simulates the enclosed tank and was constructed entirely of 8-noded brick elements. One half of this model is shown in Figure 6. The results of this system (file name PRICC) are also shown in Table 6. The heading abbreviated "Ana." in the table indicates the analytic values predicted by Price.

The results from the infinite fluid element model show reasonable correlation to the experimental values and are closer to the experimental values than the analytic results predicted by Price. This result is very encouraging given the simplicity of the model as compared to the full tank model with the accompanying reduced size of the matrices. The results from this analysis are actually as accurate as Price's analytic results for the finite depth tank.

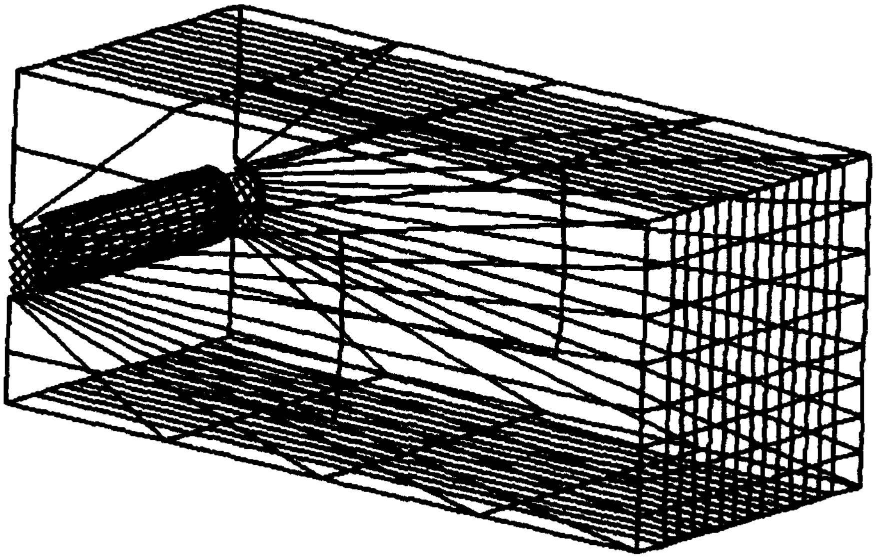

Figure 6: Half Section of Tank Fluid Model

Harmonic		Exp.	Depth = 1.6m		Depth = ∞	
N	M		Ana.	VAST	Ana.	VAST
end	cap	–	75.3	113	91.8	85.3
end	cap	–	83.3	115	101	85.3
2	1	96	90.7	148	106	90.5
3	1	107	114	168	123	115
4	1	199	245	321	253	238
3	2	214	261	364	266	266
4	2	239	–	383	–	301
2	2	338	–	508	–	340
4	3	341	–	–	–	–

Table 6: Natural Frequencies of Price Test Cylinder (in water)

The results from the full tank model are quite disappointing. The code overpredicts the experimental values significantly. Upon further investigation of these results, the poor performance was due to the wrong boundary conditions being applied at the tank walls. It was assumed that applying a free surface boundary condition at this location would suffice, but this did not properly model the physical situation. As a result, unless the tank could be included as a part of the structural model, the COUPLE/VAST combination cannot be used to solve this problem.

4 Conclusions and Recommendations

To assist in the verification of the COUPLE/VAST suite of computer codes, two unstiffened right cylinders were analysed and the results compared to those published in papers by G.C. Everstine and W.G. Price, *et al.*

Both comparisons were very close for the in-air analyses done with the VAST suite of codes. All three VAST analyses yielded results within a few percentage points of those published by Everstine, with the exception of the endcap modes which were not clearly identified in his discussion. Similarly, the two VAST analyses matched the experimental results produced by Price extremely well, with little loss in accuracy when using 4-noded shell elements versus the more capable 8-noded isoparametric elements.

For the submerged cylinder, the COUPLE/VAST combination produced results which differed significantly from those documented by Everstine, but without experimental validation, it is not possible to determine the cause of the differences. With the axisymmetric analysis, it was determined that further effort needs to be applied to the COUPLE code to allow for full harmonic analysis of axisymmetric models.

Using infinite elements, the COUPLE/VAST combination predicted the natural frequencies of the cylinder submerged in a tank as well as the analytic method used by Price which accounted for the depth of the tank. Difficulties were encountered when attempting to account for the tank walls using an exact fluid model of the tank in COUPLE. The improper boundary conditions resulted in the predicted frequencies being about 50 percent too large.

While these analyses pointed out some shortcomings of the COUPLE/VAST suite of computer codes, overall the results were encouraging. The in-air natural frequencies were predicted accurately and, in both cases, reasonably accurate predictions were made of the submerged cylinder resonances. It is readily apparent that further validation studies are required, particularly those which involve experimental results. Recent tests conducted by DREA with a ring-stiffened cylinder should go some way towards satisfying this requirement.

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